Delegating Optimal Monetary Policy Inertia: Inflation, Output Gap Growth or Nominal Income Growth Targeting?*

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Abstract. This paper shows that optimal delegation to an independent central bank with a different loss function than the societal one alleviates the stabilization bias and exactly replicates the timeless-optimal commitment equilibrium under discretion in a forward-looking model. We propose a general linear-quadratic method to solve for the optimal delegation parameters that generate the optimal amount of inertia in a Markov perfect equilibrium. The general framework nests a variety of delegation schemes that can be designed optimally: state-contingent inflation and output gap contracts, and inflation, output gap growth or nominal income growth targeting . Notably, all delegation schemes are time consistent.

JEL codes: *E31, E52, E61, C61, C73.*

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In recent models of monetary policy analysis, shocks that generate a trade-off between inflation and output stabilization are suboptimally accommodated by a central bank that acts under discretion, i.e. without taking into account the effect of its choices on private agents' expectations[1]. When such a (transitory) *cost-push* shock, hits the economy, a discretionary central bank lets inflation increase and output gap fall on impact, the variables returning thereafter to their steady-state levels. If the central bank were able to commit, i.e. take into account the influence of its actions on expectations, it could achieve a better outcome in that it would smooth the response of output gap and inflation (a feature that Woodford, 2003a has called *optimal monetary policy inertia*). And if the central bank had the technology to commit to the optimal policy, this policy would also be time-consistent (timeless perspective)[2]. Cost-push shocks are important in recent monetary policy models insofar as they help explain specific historical episodes like the 'Great Stagflation' (inflation coupled with recessions) of the 1970s, and more generally play a prominent role in explaining macroeconomic fluctuations in rich, empirically estimated dynamic stochastic general equilibrium models of the type used e.g. by Smets and Wouters (2007).

Despite the focus of a recent and growing literature on delegation of monetary policy as a way to circumvent this problem, no solution has yet been proposed that *completely eliminates* the stabilization bias and hence leads to the *exact* implementation of the timeless-optimal policy when the central bank acts discretionarily (except in some very special cases). Drawing on the pioneering work by Thompson (1981), Barro and Gordon (1983, footnote 19), Rogoff (1985), and Canzoneri (1985), a recent literature has investigated whether a central bank acting under discretion can achieve an outcome that is closer to the optimal, commitment equilibrium by appropriately changing the objective function of the central bank (i.e. delegating). In particular, Woodford (1999, 2003a), Jensen (2002), Walsh (2003) and Vestin (2006) have argued that various delegation schemes (interest rate smoothing, nominal income growth, output gap growth and price level targeting, respectively) can induce inertia and hence *improve upon* the discretionary equilibrium.

This paper tries to fill this gap by focusing on delegation as a way to *exactly replicate* the timeless-optimal commitment outcome. We draw on earlier literature that studied delegation as a way to solve other distortions, present in an earlier class of models of monetary policy analysis of the Barro-Gordon type (based on a Lucas supply function). These distortions include the average inflation bias present when the central bank targets a level of output that is higher than the socially desirable one; the state-contingent inflation bias occurring when a lagged term appears in the supply function to capture output (employment) persistence (Lockwood 1997, Svensson 1997); and the stabilization bias also present in those earlier models, that refers to the suboptimal reaction to supply shocks in the discretionary equilibrium. It should be noticed that despite this (perhaps unfortunate) coincidence of labels, the stabilization bias emphasized by that earlier literature pertains to volatilities of inflation and output and is very different from the stabilization bias present in forward-looking models described above. Most notably, the stabilization bias in

[1]Clarida, Gali and Gertler (1999) and Woodford (2003b), i.a., provide an exhaustive exposition of recent sticky-price models used for monetary policy analysis in general and of the distinction between commitment and discretionary equilibria in that framework.

[2]Kydland and Prescott (1977) is the classic reference for time inconsistency issues.

backward-looking models refers to the fact that in the discretionary equilibrium output volatility is too low, whereas inflation volatility is too high, relative to the commitment case. Instead, the stabilization bias in forward-looking models pertains most prominently to the lack of inertia induced by the policy response under discretionary policymaking.

Within that earlier class of models, a variety of delegation schemes have been proposed that exactly implement the commitment optimum when the central bank operates under discretion. This includes performance contracts (Walsh 1995, Persson and Tabellini 1993), inflation targeting (Svensson 1997), and nominal income growth targeting (Beetsma and Jensen 1999). This paper is different from earlier 'optimal delegation' papers in that it focuses on a different distortion (stabilization bias induced by forward-looking behavior). It is different from papers studying delegation as a means to improve upon the discretionary equilibrium in that it focuses on optimal delegation, i.e. on exact implementation of the timeless-optimal commitment equilibrium[3]. To the best of my knowledge, this focus on exact implementation is novel.

Throughout the paper, I focus on the simplest possible, fully-forward-looking model which features no endogenous persistence of the sort induced by the presence of lagged values of endogenous variables in structural equations (the Phillips curve and the IS curve), and no interest rate stabilization motive. This is done fro two reasons. First, this assumption allows solving for the delegation parameters analytically and contributes to an intuitive understanding of the nature of optimal delegation. Second, it isolates the role of the endogenous persistence induced under optimal commitment and hence allows a better understand the nature of optimal delegation that addresses this, the basic source of distortions in the discretionary equilibrium. Importantly, even in this simple model, the delegation schemes that induce the optimal amount of inertia turn out to be rather complex.

I start by studying one delegation schemes that is akin to the inflation contracts studied by Walsh (1995), and more precisely to the extension to state-contingent contracts studied by Lockwood (1997) and Svensson (1997). In the forward-looking New Keynesian model, the optimal contract is written over both inflation and output gap and is state contingent in the sense that marginal penalties/rewards depend upon the relevant state variable (lagged output gap); in addition, I find that the optimal weight placed on output stabilization needs to be different from that in the social loss function. This institutional arrangement bears little resemblance to policy regimes observed in practice, but it serves as a useful tool for an intuitive understanding of how delegation can impart the optimal degree of inertia.

Next, I propose a fully general method to solve for optimal delegation within the class of linear-quadratic policy problems concerned, method which nests a wide variety of delegation schemes; I then focus on particular cases that resemble real-life policy regimes and compare easily to delegation schemes studied by others. I solve for the optimal delegation parameters for a few examples including (combinations of) inflation targeting (Svensson 1997), speed limit policies or output gap growth targeting (Walsh 2003), nominal income growth targeting (Hall and Mankiw, 1994;

[3]Earlier studies have found that the timeless-optimal commitment equilibrium can be implemented by delegation in special cases. Namely, output gap growth targeting works if the central bank is fully myopic (its discount factor is zero, Walsh, 2003), interest rate smoothing if the slope of the Phillips curve is zero (Woodford, 1999) and price-level targeting if shocks have zero persistence (Vestin, 2006).

Beetsma and Jensen, 1999; Jensen, 2002) and inflation contracts (Walsh, 1993; Lockwood, 1995; Svensson, 1997).

Section 1 solves for the commitment and discretion equilibrium in a forward-looking model and reviews the stabilization bias problem. Section 2 starts by a simple example of a state-contingent contract on inflation and output gap and contains a numerical simulation of the marginal penalties necessary to implement timeless-optimal policy. Section 3 studies a fully general solution method for the optimal delegation problem within the linear-quadratic class. Section 4 uses the general method to find the optimal delegation parameters for a variety of policy regimes, including inflation targeting, speed limit policies and nominal income growth targeting. Section 5 concludes.

1. The Distortion: Stabilization Bias and Optimal Inertia in a Forward-Looking Model

This Section sets the scene by briefly reviewing the stabilization bias problem in the simplest version of the forward-looking sticky price model; A detailed analysis can be found in Clarida et al (1999) and Woodford (2003b). At the core of this model lies an inflation dynamics equation, or New Keynesian Phillips curve, derived under the assumption that a constant fraction of monopolistically competitive firms are unable to set prices in every period. This equation links current inflation π_t to its expected value $E_t\pi_{t+1}$, output gap x_t (defined as deviations of output from its efficient, flexible-price level). A stochastic disturbance u_t hits this equation: these are cost-push shocks introducing a wedge between marginal cost and output gap, and hence creating an output-inflation stabilization trade-off[4].

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t + u_t, \tag{1.1}$$

where β is the discount factor of firms and households and κ, the 'slope' of the Phillips curve, is a function of underlying preference and technology parameters.

The second equation that determines equilibrium in this model is the Euler equation for output, or the IS curve, and comes from the household's portfolio decision combined with the goods market clearing condition, linking output gap growth with ex-ante real interest rates $i_t - E_t\pi_{t+1}$ (where i_t is the central bank's instrument, a short-term nominal interest rate) and the exogenously-determined efficient level of interest rates r_t^n:

$$x_t = E_t x_{t+1} - \sigma\left[i_t - E_t\pi_{t+1} - r_t^n\right], \tag{1.2}$$

where σ is the elasticity of intertemporal substitution. Throughout this paper, I will focus on policy problems in which there is no special role for interest rate stabilization in the central bank's objective. The Euler equation (1.2) can hence be regarded as determining residually the level of interest rates i_t that is consistent with the optimal paths for inflation and output gap solved for without appealing to (1.2). Therefore, I will ignore this equation for the remainder of the analysis and think of the central bank as choosing the paths of output gap and inflation directly according to some welfare criterion.

[4]Since shocks to technology, preferences and government spending simply determine the efficient level of output (and interest rates) and will generate no tradeoff, no stabilization bias, and are ultimately irrelevant for the problem studied here, one can think of these as being completely arbitrary. This would not be the case if the central bank had an interest-rate smoothing objective as in Woodford (1999, 2003).

In this model, Woodford (2003b, Ch. 6) shows through second-order approximations to the utility function of the representative household that the welfare-relevant objective of the central bank is to minimize the present discounted sum of future squared deviations of inflation and output gap (defined as deviations of output from a notional efficient level in which prices are flexible and no cost-push shocks occur)[5]; namely, the loss function of the central bank is (proportional each period to):

$$L_t = \frac{1}{2}\left[\pi_t^2 + \lambda x_t^2\right].$$

In the simplest version of the model considered here, the weight on output gap stabilization is given by $\lambda = \kappa/\epsilon$, where κ is the slope of the Phillips curve and $\epsilon > 1$ is the elasticity of substitution between the differentiated goods entering the consumption basket of households and produced by monopolistically-competitive firms. It should be noted for further use that monopolistic competition ($\epsilon > 1$) implies that $\lambda > \kappa$.

A central bank acting under **discretion** will choose the paths of inflation and output gap to minimize the discounted sum of future losses:

$$\min_{x_t,\pi_t}\left[L_t + E_t\sum_{i=0}^{\infty}\beta^i L_{t+1+i}\right], \text{ s.t. (1.1).} \tag{1.3}$$

Under discretion, the central bank solves the problem (1.3) by taking terms involving private sector expectations as given: namely, $\beta E_t \pi_{t+1}$ in (1.1) and the second term in (1.3) are treated parametrically. The solution is the targeting rule under discretion:

$$\pi_t^d + \frac{\lambda}{\kappa}x_t^d = 0 \tag{1.4}$$

The dynamics of inflation and output are found by substituting (1.4) into the inflation equation (1.1):

$$\pi_t^d = \frac{\lambda}{\kappa^2 + \lambda\left(1 - \beta\rho_u\right)}u_t; \quad x_t^d = -\frac{\kappa}{\kappa^2 + \lambda\left(1 - \beta\rho_u\right)}u_t. \tag{1.5}$$

There is no endogenous persistence under discretion: all inertia in the dynamics of inflation and output gap come from exogenous persistence in the shock process.

Minimization of the loss function under **commitment** implies taking into account the influence on private sector expectations, and hence minimizing the whole intertemporal objective (1.3), taking the whole sequence of (1.1) as a dynamic constraint at every date. The solution is obtained by attaching a (sequence of) Lagrange multipliers to the (sequence of) dynamic constraint(s) (1.1); upon elimination of the Lagrange multipliers the first-order condition can be written as an *optimal targeting rule*:

$$\pi_t^c + \frac{\lambda}{\kappa}\left(x_t^c - x_{t-1}^c\right) = 0. \tag{1.6}$$

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[5]This approximation holds under the assumption that lump-sum taxes are available to finance a subsidy to sales that fully offsets the distortion resulting from monopolistic power in the market for goods. Without this assumption, the loss function features an extra linear term which creates a 'classical' inflation bias problem. I abstract from this complication here as delegation-based solutions to this problem have been extensively studied, as reviewed in the Introduction.

It should be noted that the targeting rule under commitment holds only starting from the period after policy was first implemented, i. e. $t \geq t_0 + 1$ if initial period is t_0, and optimality additionally requires that in the first period $\pi^c_{t_0} + \frac{\lambda}{\kappa} x^c_{t_0} = 0$.However, this policy is not time-consistent due to the arbitrariness of the initial period in which policy is chosen: optimal policy in period t_0+1 is not a continuation of optimal policy in period t_0.

A time-consistent policy, labeled *optimal policy from a timeless perspective* is implemented if the central bank commits to follow the targeting rule (1.6) for any date starting from (and *including*) the date at which policy is chosen t_0(see Proposition 7.15 in Woodford, 2003b). This policy is timeless optimal because it is the policy the central bank "would have wished to commit itself to at a date far in the past." And it is time consistent because in every period t the central bank commits to the same policy rule, since the previous period's commitment plan is not only optimal at that arbitrary date.

However, implementation of this policy is not granted for it still requires a commitment technology of the type necessary to make the central bank commit to deliver (1.6) in every period: as McCallum (1995) eloquently put it when criticizing delegation-based solutions to the average inflation bias problem: 'if a commitment technology does not exist, then it does not exist'. Optimal delegation as studied in this paper can act precisely as a substitute for this type of commitment, differently from its role in the older, Barro-Gordon type models, in which it acts as a solution to the time consistency problem itself. This is an important point, because it implies that the delegation schemes that we consider are less subject to the McCallum (1995) critique pertaining to solutions of the inflation bias problem. Since the equilibrium that our delegation schemes are purported to implement is time-consistent, the optimal delegation scheme in any period will be a continuation of optimal delegation in a previous period, and hence there would be no incentives making the principal wish to change the delegation scheme once expectations have been formed.

Optimal policy implies inertia because in this model current inflation depends on future expected inflation. Substitution of (1.6) into the Phillips curve (1.1) gives the equilibrium outcomes in terms of inflation and output under commitment and illustrates the -by now- well-known stabilization bias present in the discretionary equilibrium (see the cited papers for an extensive discussion).

$$\beta E_t x^c_{t+1} - \left(1 + \beta + \frac{\kappa^2}{\lambda}\right) x^c_t + x^c_{t-1} = \frac{\kappa}{\lambda} u_t. \tag{1.7}$$

The characteristic roots μ_1 and μ_2 are the roots of the characteristic polynomial $J(\mu)$:

$$J(\mu) = \beta\mu^2 - \left(1 + \beta + \frac{\kappa^2}{\lambda}\right)\mu + 1 = 0.$$

Since $J(0) = 1 > 0$, $J(1) = -\kappa^2/\lambda < 0$ and $J(-1) = 2(1+\beta) + \kappa^2/\lambda > 0$, it follows that $0 < \mu_1 < 1 < \mu_2$, which implies a unique bounded solution given by:

$$x^c_t = \mu_1 x^c_{t-1} - \frac{\kappa}{\beta\lambda} \sum_{j=0}^{\infty} \mu_2^{-j-1} E_t u_{t+j}$$

For an AR(1) process for the cost-push shock, this implies the equilibrium laws of motion for output gap and inflation:

$$\begin{aligned} x_t^c &= \omega_x^c x_{t-1}^c + \omega_{xu}^c u_t,\ \omega_x^c \equiv \mu_1; \omega_{xu}^c \equiv -\frac{\kappa}{\beta\lambda}\frac{1}{\mu_2-\rho_u} \\ \pi_t^c &= \omega_\pi^c x_{t-1}^c + \omega_{\pi u}^c u_t,\ \omega_\pi^c \equiv \frac{\lambda}{\kappa}(1-\mu_1); \omega_{\pi u}^c = -\frac{\lambda}{\kappa}\omega_{xu}^c. \end{aligned} \tag{1.8}$$

To illustrate the role of commitment in generating inertia, or endogenous persistence, consider the case of a purely transitory shock, $\rho_u = 0$. Under discretion, output gap and inflation will immediately return to their zero steady-state value after the period when the shock hits, since the central bank reoptimizes every period. Under commitment, this is no longer true: purely transitory shocks will imply a persistently higher inflation and persistently lower output. By committing to such a policy for future periods, the central bank obtains a lower impact response of inflation than under discretion, and a smaller fall in the output gap - that is, the central bank faces a better trade-off under inflation and output stabilization than under discretion. This happens because current inflation depends on future expected inflation: a persistent fall in the output gap 'buys' a smaller increase in inflation today because it implies a fall in expected future inflation. Otherwise put, under discretion there is a stabilization bias due to not internalizing the effects of future output gap variation on current inflation through inflation expectations (see i.a. Woodford, 2003a for an extensive discussion).

2. Optimal Delegation

The solution proposed by this paper to alleviate the stabilization bias problem is based on a very simple idea: modify the loss function under discretion in order to induce the central bank implement the commitment solution. In doing so, I draw on a large and important literature reviewed in the introduction that studied institutional design as a solution to various distortions present in discretionary monetary policymaking. In the most general case, delegation amounts to choosing a central bank that has preferences given by a loss function of the form:

$$L_t^b(.,\Theta) = L_t + F(\pi_t, x_t, x_{t-1}, u_t; \Theta),$$

where superscript b stands for 'bank' and the additional term $F(.)$ is a function of all relevant macroeconomic variables, shocks, and a matrix Θ of new parameters θ that are to be chosen optimally.

Restrictions can be placed upon the delegation function $F(.)$ by the very nature of the problem to be solved. First, the difference between the discretion and commitment first order conditions (1.4) or (1.6) involves only a linear term in past output gap, and no constant. Therefore, we restrict F to be a quadratic form[6]. Moreover, since the stabilization bias comes from an absence of an inertial term in the discretionary equilibrium, optimal delegation should induce this history-dependence: that is why the delegated loss function should made be dependent

[6] A linear term would be needed if a linear term appeared in the societal loss function (for example if subsidies were not available to eliminate the steady-state monopolistic distortion making output too low, see Woodford, 2003 Ch. 6). That is because in that case an average inflation bias would occur similar to the inflation bias present in Barro-Gordon models, and a linear inflation contract would alleviate that problem (Walsh, 1995). Since this problem (and its solution) are well understood I will abstract from it here in order to focus on what is novel.

upon lagged output gap, so that the latter appears in the new first-order condition. On the contrary, since there is no term depending upon the shock u_t directly in the first-order condition, the shock should not appear as such in the delegated loss function.

The optimal delegation problem amounts to choosing those delegation parameters $\Theta = \Theta^*$ that lead to implementation of the commitment equilibrium outcomes under discretion and can be solved as follows. The first order condition of the discretionary optimal policy problem under the modified loss function L^b will involve inflation, output gap and the lagged value of output gap. Through the dynamic constraint represented by the Phillips curve, this will be represented as a second-order difference equation for output gap, just as in the commitment equilibrium. In the discretionary equilibrium, however, the coefficients of the equation will depend on the delegation parameters Θ. Finding optimal delegation parameters Θ^* will then involve applying the method of undetermined coefficients by equating the coefficients appearing in the timeless-optimal solution with those in the delegated discretionary case.

Since we have argued that optimal delegation should necessarily involve lagged output gap, which becomes an endogenous state variable, the definition of discretionary equilibrium will depend upon the assumption made concerning the treatment of expectations by the central bank. Indeed, as emphasized by McCallum and Nelson (2000) and Walsh (2003), there are two possible definitions of the discretionary equilibrium. In a first definition, as in the standard case without endogenous state variables, the central bank ignores the effect of its actions on private expectations altogether, despite the presence of an endogenous state variable. In a second version, the central bank recognizes that past value of the output gap (which are instead influenced by its past choices) act as endogenous state variables, and hence help the private sector predict its future actions. Contrary to the first version, the central bank does not take expectations as given, but takes into account the effect of its actions on private decision rules. Throughout this paper, I will use the second version, i.e. the **dynamic programming, Markov perfect solution** and simply note that it nests the first version as should become clear below. I first provide an example of an optimal delegation scheme that is transparent and allows a clear understanding on the solution mechanism and return to the general case in the next section.

2.1. An Optimal Contract. Suppose that monetary policy is delegated to a central bank with the following per-period loss function such that:

$$L_t^b = \frac{1}{2}\left[\lambda^b x_t^2 + \pi_t^2 + 2c_\pi x_{t-1}\pi_t + 2c_x x_{t-1}x_t\right], \tag{2.1}$$

which incorporates a penalty/reward for additional inflation and output gap, and that the marginal penalty depends upon the state variable, i.e. the past value of output gap. Furthermore, the weight on output stabilization is allowed to differ from the social one. This resembles loss functions considered i.a. by Svensson (1997). There are three new parameters[7] $\Theta \equiv \{\lambda^b, c_\pi, c_x\}$, whose optimal values are found in the following Proposition.

[7] There is a good reason why the number of parameters is three, which should become clearer below: since the solution method relies upon the method of undetermined coefficients and the difference equation governing the solution in the commitment case is second-order (and hence features three coefficients), we need three free parameters.

PROPOSITION 1. *The Markov-perfect equilibrium value of inflation and output gap occurring if the central bank minimizes the delegated loss function (2.1) are identical to the timeless-optimal commitment solution (1.8) if and only if the delegation parameters are given by:*

$$\lambda^{b*} = \frac{\lambda}{\kappa}\gamma\left(1+\frac{\beta\lambda}{\kappa\gamma+\lambda}\right);\; c_\pi^* = -\frac{\lambda\gamma}{\kappa\gamma+\lambda};\; c_x^* = c_\pi^*\frac{\lambda}{\kappa} \tag{2.2}$$

The proof of this Proposition (and the subsequent ones) is useful for an intuitive understanding of the mechanism governing optimal delegation, and is hence included in the main text. The Markov-perfect, time consistent equilibrium is found by utilizing standard dynamic programming techniques. The central bank's value function $V(x_{t-1};u_t)$ needs to satisfy the Bellman equation:

$$\begin{aligned} V(x_{t-1};u_t) &= \min_{x_t,\pi_t}\frac{1}{2}\left[\lambda^b x_t^2+\pi_t^2+2c_\pi x_{t-1}\pi_t+2c_x x_{t-1}x_t+\beta E_t V(x_t;u_{t+1})\right] \\ \text{(2.3)}\quad s.t.\ \pi_t &= \beta E_t\left[\pi_{t+1}(x_t,u_{t+1})\right]+\kappa x_t+u_t. \end{aligned}$$

A solution to this problem given the state $(x_{t-1};u_t)$ is a pair of state-contingent decision rules for output and inflation $x^b(x_{t-1},u_t), \pi^b(x_{t-1},u_t)$ and a value function $V^b(x_{t-1};u_t)$. Since the problem is a linear-quadratic one, the value function will be quadratic and the decision rules will be linear, with coefficients depending on delegation parameters:

$$\begin{aligned} x^b(x_{t-1},u_t) &= \omega_x(\Theta)x_{t-1}+\omega_{xu}(\Theta)u_t \\ \pi^b(x_{t-1},u_t) &= \omega_\pi(\Theta)x_{t-1}+\omega_{\pi u}(\Theta)u_t, \end{aligned} \tag{2.4}$$

Furthermore, since the term in square brackets in (2.3) is quadratic, it achieves a unique minimum if and only if it is convex, in which case the optimum is characterized by the necessary first-order condition. Because the function is quadratic, it is globally convex if and only if the second-order condition is satisfied. A solution satisfying the first- and second-order conditions is both necessary and sufficient for optimality.

To solve the optimal delegation problem, however, we do not need to solve for the decision rules (2.4) explicitly. Instead, recall that the optimal delegation problem consists of finding those delegation parameters Θ such that the laws of motion are the same as those obtained under timeless commitment (1.8). There are two ways to approach this problem. First, one could employ brute force, solve for the laws of motion (2.4) and find the parameters Θ^* by undetermined coefficients, i.e. by solving $\omega_j(\Theta^*) = \omega_j^c$ for the delegation parameters.

An equivalent and perhaps more elegant solution is to conjecture that optimal delegation is in place, and so that the laws of motion (1.8) and (2.4) are identical $\omega_j(\Theta) = \omega_j^c$. The optimal delegation parameters Θ^* are then found by using the restrictions derived from the fact that the first-order condition of the central bank's problem (2.3) should necessarily be satisfied for the conjectured timeless-optimal commitment solution. Furthermore, one needs to verify that for the found values Θ^* the second order condition is also satisfied, which ensures that the timeless-optimal commitment solution is the only equilibrium of the central bank's problem under delegation. This is the method used in this paper.

The first order condition for (2.3) is[8]:

$$\lambda^b x_t + \pi_t \left(\kappa + \beta \frac{\partial}{\partial x_t} E_t \left[\pi_{t+1}\left(x_t, u_{t+1}\right)\right]\right) + c_\pi x_{t-1} \left(\kappa + \beta \frac{\partial}{\partial x_t} E_t \left[\pi_{t+1}\left(x_t, u_{t+1}\right)\right]\right) + \quad : \\ + c_x x_{t-1} + \beta E_t \frac{\partial}{\partial x_t} V\left(x_t; u_{t+1}\right) = 0$$

The Envelope theorem implies:

$$\frac{\partial}{\partial x_{t-1}} V\left(x_{t-1}; u_t\right) = c_\pi \pi_t + c_x x_t,$$

so the first-order condition becomes:

$$\lambda^b x_t + \pi_t \left(\kappa + \beta \frac{\partial}{\partial x_t} E_t \left[\pi_{t+1}\left(x_t, u_{t+1}\right)\right]\right) + \quad : \\ + c_\pi x_{t-1} \left(\kappa + \beta \frac{\partial}{\partial x_t} E_t \left[\pi_{t+1}\left(x_t, u_{t+1}\right)\right]\right) + c_x x_{t-1} + \beta E_t \left[c_\pi \pi_{t+1} + c_x x_{t+1}\right] = 0. \tag{2.5}$$

Recall that we are looking for those delegation parameters that will lead to implementation of the timeless optimal commitment, whereby the laws of motion are (1.8) and the first order condition for commitment (1.6) holds. Therefore, expectations will be formed according to:

$$\begin{aligned} E_t x_{t+1} &= \omega_x^c x_t + \omega_{xu}^c E_t u_{t+1} \\ E_t \pi_{t+1} &= \omega_\pi^c x_t + \omega_{\pi u}^c E_t u_{t+1}. \end{aligned} \tag{2.6}$$

Substituting in (2.5) the first derivative of expected inflation using (2.6) and inflation using the first order condition under commitment (1.6), and grouping coefficients, we obtain:

$$\beta \left(c_x - c_\pi \frac{\lambda}{\kappa}\right) E_t x_{t+1} + \left(\lambda^b - \frac{\lambda}{\kappa}\gamma + \beta c_\pi \frac{\lambda}{\kappa}\right) x_t + \left(\frac{\lambda}{\kappa}\gamma + c_\pi \gamma + c_x\right) x_{t-1} = 0$$

where I used the notation $\gamma \equiv \kappa + \beta \frac{\partial}{\partial x_t} E_t \left[\pi_{t+1}\left(x_t, u_{t+1}\right)\right] = \kappa + \beta \omega_\pi^c$. The optimal delegation parameters are found by noticing that this equation evaluated at the conjectured solution $\left(x_t^c, \pi_t^c\right)$ should be an identity; therefore, all coefficients should be zero, and the optimal delegation parameters are found, in case $\beta > 0$, as in Proposition 1. In the Appendix, I verify that under the loss function implied by the optimal contract (2.2), the first-order condition of the Markov perfect equilibrium indeed coincides with that of the timeless-optimal commitment equilibrium (1.6); this verification can be done identically for all delegation schemes considered below.

All we have proved up to now is that under the delegation parameters Θ^*, the timeless-optimal commitment solution satisfies the first-order condition of the central bank's problem under discretion. In order to prove that this is the unique equilibrium occurring, we need to show that the second-order condition is satisfied. This is important, because there exist delegation schemes in which the first-order condition holds, but the second-order condition fails; Two examples of such delegation schemes are provided in Appendix B. The second-order condition is:

$$\lambda^{b*} + \gamma^2 + \beta c_\pi^* \omega_\pi^c + \beta c_x^* \omega_x^c \geq 0.$$

[8]Notice that our method nests the 'pure discretion' case studied by McCallum and Nelson (2000) and Walsh (2003) when an additional constraint $\frac{\partial}{\partial x_t} E_t \left[\pi_{t+1}\left(x_t, u_{t+1}\right)\right] = 0$ is added, implying that the central bank ignores the effect of its actions on expectation.

Substituting the optimal delegation parameters found above, and the transition coefficients ω_π^c, ω_x^c found in (1.8) we see that the condition is indeed satisfied: $\frac{\lambda}{\kappa}\gamma + \gamma^2 \geq 0$.

Why does the contract found in Proposition 1 induce the optimal amount of inertia? Consider a bank operating under the contract (the loss function (2.1)) and facing a one-time cost-push shock. Since the bank chooses policy discretionarily, it will on impact let inflation increase and contract output gap. Since this is last period's output gap from next period's viewpoint, the central bank's incentives are changed: a further contraction in output gap (as required by optimal policy) is rewarded if $c_x^* < 0$ and a further increase in inflation is costly if $c_\pi^* < 0$. The extent to which (given a past contraction) future inflation is costly relative to how beneficial it is to let the contraction persist is given by the parameter governing the original, social trade-off between output and inflation λ/κ. Therefore, the optimal amount of inertia can be replicated by the optimal choice of the parameters governing the extent to which (given a negative output gap today) future inflation is costly and future contractions are beneficial.

Figure 1 plots the macroeconomic outcomes, i.e. impulse responses of output gap and inflation, obtained by solving a parameterized version of the model in response to a one-time cost-push shock, under two scenarios: pure discretion (dashed red line) and optimal delegation (which is identical to the timeless optimal commitment). In the latter case, we also plot the evolution of the optimal marginal penalties on inflation and output gap, $c_\pi^* x_{t-1}$ and $c_x^* x_{t-1}$. The parameterization is entirely standard, namely the discount factor is $\beta = 0.99$, a slope of the Phillips curve of $\kappa = 0.4292$ (consistent in the simplest version of the underlying model with an average price duration of one year and a labor supply elasticity of 0.25) and an elasticity of substitution between differentiated goods of $\epsilon = 6$, leading to a steady-state gross markup of 1.2. For these parameter values, the optimal delegation parameters are: $\lambda^{b*} = 0.1138$; $c_\pi^* = -0.1282$; $c_x^* = -0.0214$. Note that these parameter values imply that the central bank is 'liberal' in the sense of Rogoff (1985), since the weight placed on output gap stabilization exceeds the societal one ($\lambda^{b*} > \lambda = 0.0715$). The figure substantiates our previous intuition, by showing how the marginal penalties necessary to support the timeless-optimal outcome impart the optimal amount of inertia that is absent in the discretionary case. Faced with penalties next period on both inflating and decreasing the output gap that depend on its actions today, the central bank increases inflation by less and decreases output gap by less compared to the case of pure discretion. In contrast to the equilibrium in the absence of delegation, it is optimal for the central bank to let the responses persist, because today's output contraction combined with $c_x^* < 0$ and $c_\pi^* < 0$ effectively induces incentives for the central bank to reduce both output and inflation tomorrow. Figure 2 repeats the same exercise for a cost-push shock with persistence 0.9.

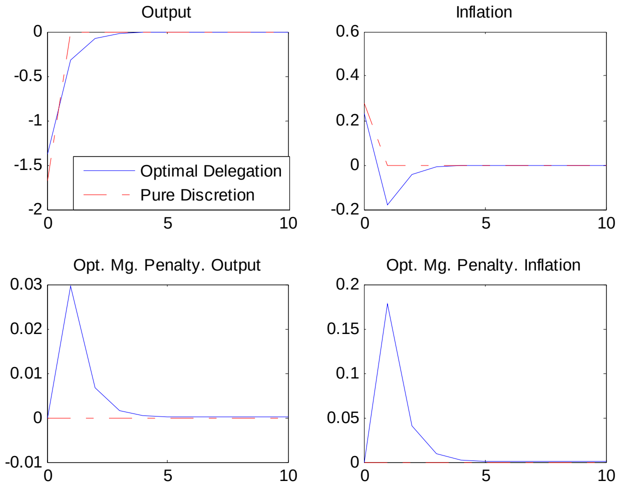

Figure 1: Responses of output gap, inflation and the marginal penalties on output and inflation to a one-time cost-push shock under two policy regimes: optimal delegation and pure discretion.

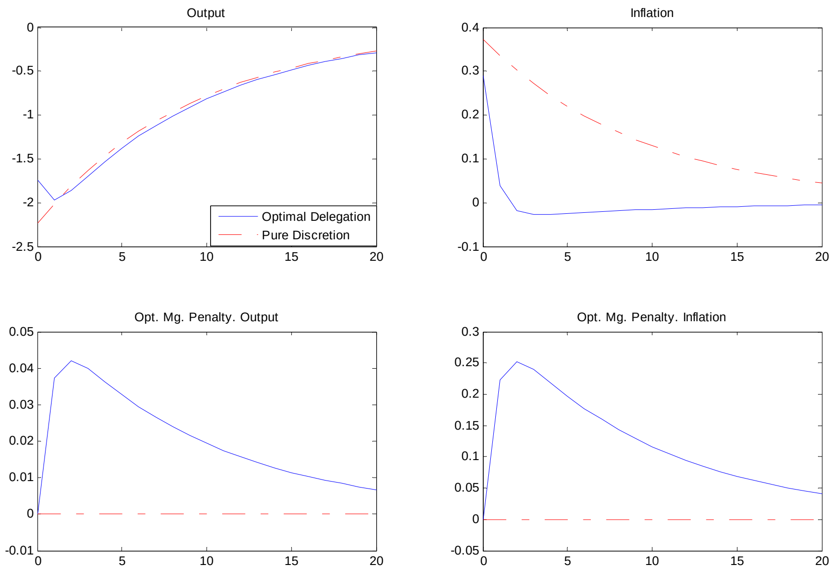

Figure 2: Responses of output gap, inflation and the marginal penalties on output and inflation to a persistent cost-push shock under two policy regimes: optimal delegation and pure discretion.

3. Optimal Delegation: A General Linear-Quadratic Method

The linear contract studied in the previous section bears little resemblance to real-life policy arrangements. However, it is useful as a benchmark for understanding the solution method of this paper, which can easily be generalized to other delegation schemes within the quadratic class. This is what this Section does, setting the scene for the next Section which considers targeting regimes that resemble actual policy arrangements. Since the philosophy of the solution method is indeed identical, I will not go into details but rather concentrate on the new elements allowed by generality.

Suppose that delegation takes the general form of adding to the societal loss function a quadratic form $Z_t^{'}\Theta Z_t$ over all relevant variables $Z_t = (\pi_t,\ x_t,\ x_{t-1})'$:

$$L_t^b = \frac{1}{2}\left[L_t + Z_t^{'}\Theta Z_t\right], \tag{3.1}$$

where Θ is the symmetric matrix of all delegation parameters, $\Theta = \{\theta_{ij}\}, i, j = 1, 2, 3,\ \theta_{ij} = \theta_{ji}$; this form nests all policy arrangements one could think of, and some examples will be provided below[9].The following Proposition provides the conditions that need to be fulfilled by delegation parameters in order to induce the optimal amount of inertia.

Proposition 2. *The Markov-perfect equilibrium value of inflation and output gap occurring if the central bank minimizes the delegated loss function (3.1) are identical to the timeless-optimal commitment solution (1.8) if and only if the delegation parameters satisfy:*

$$\begin{aligned} \theta_{xs}^* &= \frac{\lambda}{\kappa}\theta_{\pi s}^* \\ \lambda + \theta_{xx}^* + \gamma\theta_{\pi x}^* + \beta\theta_{ss}^* &= -\left(\gamma + \frac{\lambda}{\kappa} + \frac{\lambda}{\kappa}\beta\right)\theta_{\pi s}^* \\ \gamma\left(1 + \theta_{\pi\pi}^*\right) + \theta_{\pi x}^* &= -\left(\frac{\kappa}{\lambda}\gamma + 1\right)\theta_{\pi s}^* \end{aligned} \tag{3.2}$$

and

$$\theta_{\pi s}^* < 0. \tag{3.3}$$

The proof is similar to that of Proposition 1: the Markov-perfect equilibrium values of inflation and output gap need to satisfy the Bellman equation:

$$\begin{aligned} V\left(x_{t-1}; u_t\right) &= \min_{x_t, \pi_t} \frac{1}{2}\left[\lambda x_t^2 + \pi_t^2 + Z_t^{'}\Theta Z_t + \beta E_t V\left(x_t; u_{t+1}\right)\right], \\ s.t.\ \pi_t &= \beta E_t\left[\pi_{t+1}\left(x_t, u_{t+1}\right)\right] + \kappa x_t + u_t. \end{aligned}$$

The first order condition is

$$\lambda x_t + \left(\kappa + \beta\frac{\partial}{\partial x_t}E_t\left[\pi_{t+1}\left(x_t, u_{t+1}\right)\right]\right)\pi_t + Z_t^{'}\Theta\nabla_{x_t}Z_t + \beta E_t\frac{\partial}{\partial x_t}V\left(x_t; u_{t+1}\right) = 0,$$

[9]Clearly, of the six parameters θ_{ij}, one could be fixed a priori by means of normalization-However, I do not impose this since not all delegation schemes I wish to consider will necesarily share the same normalization; it is hence more transparent to let all parameters free to start with.

where $\nabla_{x_t} Z_t = \left(\kappa + \beta \frac{\partial}{\partial x_t} E_t \left[\pi_{t+1}(x_t, u_{t+1})\right],\ 1,\ 0\right)'$ is the gradient of Z_t. The Envelope theorem implies:

$$\frac{\partial}{\partial x_{t-1}} V(x_{t-1}; u_t) = Z_t^{'} \Theta e_3,$$

where e_j denotes a vector of appropriate dimension whose all elements are zero, except for the jth element which is 1; accordingly $e_3 = (0,0,1)$.The first-order condition becomes:

$$\lambda x_t + \left(\kappa + \beta \frac{\partial}{\partial x_t} E_t \left[\pi_{t+1}(x_t, u_{t+1})\right]\right) \pi_t + Z_t^{'} \Theta \nabla_{x_t} Z_t + \beta E_t Z_{t+1}^{'} \Theta e_3 = 0$$

Recall that, as in the special case studied in the previous section, we are looking for those delegation parameters that will lead to implementation of the timeless optimal commitment, whereby the laws of motion are (1.8) and the first order condition for commitment (1.6) holds. Therefore, expectations will be formed according to (2.6), which when taken into account in the first-order condition implies[10]:

$$Z_t^{'} \left[(\gamma,\ \lambda,\ 0)' + \Theta (\gamma,\ 1,\ 0)' \right] + \beta E_t Z_{t+1}^{'} \Theta (0,0,1)' = 0$$

At this stage, it is useful to rewrite the equation explicitly in terms of the original variables:

$$\begin{aligned} (\gamma + \gamma\theta_{\pi\pi} + \theta_{\pi x}) \pi_t + (\lambda + \gamma\theta_{\pi x} + \theta_{xx} + \beta\theta_{ss}) x_t + & : \\ + (\gamma\theta_{\pi s} + \theta_{xs}) x_{t-1} + \beta\theta_{\pi s} E_t \pi_{t+1} + \beta\theta_{xs} E_t x_{t+1} &= 0 \end{aligned}$$

Substitute the timeless optimal first order condition (1.6):

$$\begin{aligned} \beta\left(\theta_{xs} - \frac{\lambda}{\kappa}\theta_{\pi s}\right) E_t x_{t+1} + + \left(\lambda + \gamma\theta_{\pi x} + \theta_{xx} + \beta\theta_{ss} + \frac{\lambda}{\kappa}\beta\theta_{\pi s} - \frac{\lambda}{\kappa}(\gamma + \gamma\theta_{\pi\pi} + \theta_{\pi x})\right) x_t & : \\ + \left(\gamma\theta_{\pi s} + \theta_{xs} + \frac{\lambda}{\kappa}(\gamma + \gamma\theta_{\pi\pi} + \theta_{\pi x})\right) x_{t-1} &= 0 \end{aligned}$$

The optimal delegation parameters are again found by noticing that this equation evaluated at the conjectured solution (x_t^c, π_t^c) should be an identity; therefore, all coefficients should be zero, and the optimal delegation parameters Θ^* are found by solving:

$$\begin{aligned} \beta\left(\theta_{xs} - \frac{\lambda}{\kappa}\theta_{\pi s}\right) &= 0 \\ \lambda + \gamma\theta_{\pi x} + \theta_{xx} + \beta\theta_{ss} + \frac{\lambda}{\kappa}\beta\theta_{\pi s} - \frac{\lambda}{\kappa}(\gamma + \gamma\theta_{\pi\pi} + \theta_{\pi x}) &= 0 \\ \gamma\theta_{\pi s} + \theta_{xs} + \frac{\lambda}{\kappa}(\gamma + \gamma\theta_{\pi\pi} + \theta_{\pi x}) &= 0 \end{aligned}$$

For $\beta > 0$, the six delegation parameters should satisfy the three restrictions provided in (3.2).

Parameters satisfying these restrictions ensure that the first-order condition occurring in the central bank's problem under delegation is identical to the one under timeless-optimal commitment. To ensure that the commitment equilibrium

[10] I use again the notation $\gamma \equiv \kappa + \beta\omega_\pi^c = \kappa + \beta \frac{\partial}{\partial x_t} E_t \left[\pi_{t+1}(x_t, u_{t+1})\right]$

is the only equilibrium occurring in the new problem under delegation, we need to ensure that the parameters Θ^* satisfy the second order condition:

$$\lambda + \left(\kappa + \beta \frac{\partial}{\partial x_t} E_t \left[\pi_{t+1}\left(x_t, u_{t+1}\right)\right]\right)^2 + \left(\nabla_{x_t} Z_t\right)' \Theta \nabla_{x_t} Z_t + \beta E_t \left(\nabla_{x_t} Z_{t+1}\right)' \Theta e_3 > 0,$$

$$\text{or } \lambda + \theta^*_{xx} + \gamma^2 \left(1 + \theta^*_{\pi\pi}\right) + 2\gamma \theta^*_{\pi x} + \beta \omega^c_{\pi} \theta^*_{\pi s} + \beta \omega^c_{x} \theta^*_{xs} + \beta \theta^*_{ss} > 0 \tag{3.4}$$

Using the properties of the optimal delegation parameters implied by the first-order condition under $\beta > 0$, (3.2), this boils down to (3.3).

A first piece of intuition as to how delegation helps to bring about the optimal degree of inertia can be obtained just by considering the second-order condition (3.3) and the first parameter restriction in (3.2). The second-order condition (3.3) implies that given a past contraction in output gap, an increase in future inflation will have to be penalized, making inflation more costly as required by optimal policy. And the first restriction in (3.2) implies that for the same past contraction in output gap, future contractions should be rewarded, for it is future contractions that allow the central bank to contain inflation today and hence face a better trade-off. The extent to which (given a past contraction) future inflation is costly relative to how beneficial it is to let the contraction persist is governed by the parameter governing the original, social trade-off between output and inflation λ/κ. Further intuition can be gained by studying particular targeting regimes.

Different delegation schemes can be modelled by imposing restrictions on the $\boldsymbol{\Theta}$ matrix. Restrictions can be obtained by noticing that θ_{xs} and $\theta_{\pi s}$ cannot be zero individually. A first illustrative example is obtained by noting that the two restrictions $\theta_{\pi x} = \theta_{ss} = 0$ and the normalization $\theta_{\pi\pi} = 0$ imply:

$$\theta^*_{\pi s} = -\frac{\lambda\gamma}{\gamma\kappa + \lambda};\ \theta^*_{xs} = \frac{\lambda}{\kappa}\theta^*_{\pi s};\ \lambda + \theta^*_{xx} = \lambda\frac{\gamma}{\kappa}\left(1 + \frac{\lambda\beta}{\kappa\gamma + \lambda}\right) > \lambda.$$

This is precisely the optimal contract found in Proposition 1.

4. Optimal Targeting Regimes

This Section uses the general conditions for optimal delegation derived in the previous section to study delegation schemes in the form of 'targeting regimes' (in the sense of Svensson, 1999a) that resemble real-life policy arrangements. Specifically, I study (combinations of) inflation targeting following Svensson (1997, 1999a), output gap growth targeting or speed limit policies following Walsh (2003) and nominal income growth targeting in the sense of Beetsma and Jensen (1999) and Jensen (2002)[11]. In each case, I postulate a loss function that describes each policy regime and map it back into the general form used in the previous section. I then derive the optimal delegation parameters of each targeting regime by exploiting the parameter restrictions derived in the general case (3.2).

Unfortunately, no targeting regime by itself can be employed to implement the timeless-commitment optimum, and the reason is that each targeting regime changes incentives only in one dimension (inflation or output gap), for a given value of past output gap. Take for example the speed limit policy studied by Walsh,

[11]Since I restrict attention to delegation schemes focusing on inflation and output gap only, the delegation schemes I consider cannot nest the interest rate smoothing proposed by Woodford (1999, 2003a) or the price level targeting regime studied by Vestin (2006). The framework could be extended to deal with these extensions at the cost of increased complexity.

whereby the central bank seeks to stabilize output gap growth, and the weight put on this objective is λ_G (which needs to be determined optimally):

$$L_t^b = \frac{1}{2}\left[\lambda_G\left(x_t - x_{t-1}\right)^2 + \pi_t^2\right].$$

Since in this case $\theta_{xs} = -\lambda_G$ and $\theta_{\pi s} = 0$, it is clear that the first restriction in (3.2) is violated and hence that the speed limit policy is not sufficient by itself for implementing optimal delegation. As already noted by Walsh (2003), there is one circumstance, however, in which speed limit policy induces optimality, and that is when the central bank if fully myopic ($\beta = 0$), which furthermore implies $\kappa = \gamma$. In that case the first restriction in (3.2) is not binding, and the remaining restrictions imply that any scheme satisfying:

$$\lambda + \theta_{xx} + \kappa\theta_{\pi x} = -\kappa\theta_{\pi s} - \theta_{xs} = \lambda\left(1 + \theta_{\pi\pi}\right) + \frac{\lambda}{\kappa}\theta_{\pi x}, \text{with } \theta_{ss} \text{ arbitrary}$$

will work. The speed limit policy with $\lambda_G = \lambda$ is just one of an infinity of delegation schemes that would work if the central bank were myopic.[12]

The same reasoning applies for the type of state-contingent inflation targets studied by Svensson (1997) in the context of a Barro-Gordon type model. Suppose that the central bank is assigned a state-contingent inflation target and a different weight on inflation stabilization than society's, namely:

$$L_t^b = \frac{1}{2}\left[\lambda x_t^2 + \alpha\left(\pi_t - \pi_t^*\right)^2\right], \text{ with } \pi_t^* = \delta x_{t-1}$$

where α and δ are the parameters to be determined[13]. Since in this case $\theta_{xs} = 0$ and $\theta_{\pi s} = -\alpha\delta$, it is again apparent that the first restriction in (3.2) is violated and hence that the inflation target by itself fails to implement optimal delegation. But again in the fully myopic case $\beta = 0$, the timeless-optimal commitment optimum is implemented by the myopic central bank if:

$$\alpha = 1, \delta = \frac{\lambda}{\kappa}.$$

Since none of the two targeting regimes would implement optimal delegation in the discounting case $\beta > 0$, and since the reason of each one's failure is precisely the absence of an inertial term that the other would imply, a natural candidate is a policy regime that combines the two.

4.1. A speed limit policy and a state-contingent inflation target. If the central bank is assigned both an output growth targeting and an inflation targeting objective, the loss function is:

$$L_t^b = \lambda x_t^2 + \lambda_G\left(x_t - x_{t-1}\right)^2 + \alpha\left(\pi_t - \pi_t^*\right)^2, \text{ with } \pi_t^* = \delta x_{t-1} \tag{4.1}$$

PROPOSITION 3. *The Markov-perfect equilibrium value of inflation and output gap occurring if the central bank minimizes the delegated loss function (4.1)*

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[12] Specifically, this occurs for $\theta_{xx} = \theta_{\pi x} = \theta_{\pi s} = \theta_{\pi\pi} = 0, \theta_{xs} = -\lambda, \theta_{ss} = \lambda$.If $\beta > 0$, Walsh shows that it may be possible to choose the weight on output gap growth stabilization λ_G such that the optimal amount of inertia is induced, but then shock stabilization is suboptimal.

[13] Differently from Svensson (1997), there is no constant term in the inflation target π_t^* because as explained at the outset I abstract from the classical (average) inflation bias problem. Note also that I implicitly used the normalization $\theta_{xx} = 0$.

are identical to the timeless-optimal commitment solution (1.8) if and only if the delegation parameters are given by:

$$\lambda_G^* = \frac{\lambda^2}{\kappa\gamma}\frac{\kappa\gamma+\lambda}{\kappa\gamma+(1-\beta)\lambda};\ \alpha^* = \frac{(\kappa\gamma+\lambda)^2}{\gamma^2(\kappa\gamma+(1-\beta)\lambda)};\ \delta^* = \frac{\lambda\gamma}{\kappa\gamma+\lambda}.$$

This proof of this proposition is immediate once one notes that for the loss function (4.1), we have the following mapping between its delegation parameters and the delegation parameters in the general case (the Θ matrix):

$$\theta_{xx} = \lambda_G; 1+\theta_{\pi\pi} = \alpha; \theta_{ss} = \lambda_G + \alpha\delta^2; \theta_{\pi x} = 0; \theta_{\pi s} = -\alpha\delta; \theta_{xs} = -\lambda_G$$

Substituting this in the parameter restrictions for optimal delegation (3.2), we obtain the optimal delegation parameters for our targeting regime[14] as in Proposition 3. Finally, since $\alpha^* > 0$ and $\delta^* > 0$, it follows that $\theta_{\pi s}^* = -\alpha^*\delta^* < 0$: the second-order condition is also satisfied, and so the timeless-optimal commitment equilibrium is the only equilibrium occurring in the delegated policy problem under discretion.

This targeting regime generates the optimal amount of inertia because, given the initial contraction in output gap required by discretionary optimization, the speed-limit component makes the output gap contraction persist, while the state-contingent inflation target makes it suboptimal for inflation to return to the steady-state immediately; instead, given last period's contraction, the inflation target becomes positive, and the loss is minimized by deflating in future periods.

4.2. Walsh (2003) meets Walsh (1995): speed limit policy and inflation contract. Optimality is similarly restored if the speed limit policy is combined with an inflation contract in the spirit of Walsh (1995) that is allowed to be state-contingent,[15] namely for a loss function of the form:

$$L_t^b = \lambda x_t^2 + \lambda_G (x_t - x_{t-1})^2 + \alpha {\pi_t}^2 + 2c_\pi x_{t-1}\pi_t \tag{4.2}$$

Proposition 4. *The Markov-perfect equilibrium value of inflation and output gap occurring if the central bank minimizes the delegated loss function (4.2) are identical to the timeless-optimal commitment solution (1.8) if and only if the delegation parameters are given by:*

$$\lambda_G^* = \frac{\lambda^2}{\kappa\gamma};\ \alpha^* = \left(\frac{\kappa}{\lambda}\gamma+1\right)\frac{\lambda}{\gamma^2};\ c_\pi^* = -\frac{\lambda}{\gamma}.$$

The proof is similar to that of Proposition 3: for the loss function (4.2), the mapping between the new delegation parameters and the Θ matrix is:

$$\theta_{xx} = \lambda_G; 1+\theta_{\pi\pi} = \alpha; \theta_{ss} = \lambda_G; \theta_{\pi x} = 0; \theta_{\pi s} = c_\pi; \theta_{xs} = -\lambda_G,$$

which substituted in the parameter restrictions for optimal delegation (3.2) delivers the optimal delegation parameters in Proposition 4. The second-order condition is satisfied since since $\theta_{\pi s}^* = c_\pi^* < 0$.

The intuition as to why this induces optimal inertia is similar to the previous targeting regime, the only difference being that given a past output gap contraction

[14]Following the same steps as in Appendix A, the skeptical reader can verify that under these delegation parameters the timeless-optimal first-order condition (1.6) occurs in the Markov Perfect equilibrium.

[15]To be precise, since the inflation contract I consider is a state-contingent one, it is more in the spirit of those proposed Lockwood et al (1995) and Svensson (1997).

the optimal incentive for the central bank policy concerning inflation is provided by making inflation costly (or indeed rewarding deflation).

4.3. Nominal income growth targeting. Beetsma and Jensen (1999) have argued that state-dependent delegation schemes have the undesirable feature that the delegation parameter changes over time, which ultimately undermines credibility and accentuates the problem identified by McCallum (1995) and Jensen (1997) and reviewed above. Therefore, these authors have proposed a state-independent delegation scheme in the form of nominal income growth targeting[16]. Jensen (2002) has further studied the merits of this targeting regime in the context of a New Keynesian model close to the one studied here. This policy regime is captured by the following loss function:

$$L_t^b = \lambda x_t^2 + \alpha \pi_t^2 + \psi \left(\pi_t + x_t - x_{t-1}\right)^2 , \tag{4.3}$$

Since for this loss function $\theta_{\pi s} = \theta_{xs} = -\psi$, it is again clear that the first restriction in (3.2) is violated and hence that this regime by itself fails to implement optimal delegation. In order for that restriction to be satisfied, the delegation scheme needs to include another free delegation parameter pertaining to the interaction between past output gap and inflation or output gap today. Two such regimes can be though of by adding to the loss function (4.3) a state-contingent inflation target or a state-contingent inflation contract respectively. In the former case the loss function becomes:

$$L_t^b = \lambda x_t^2 + \alpha \left(\pi_t - \delta x_{t-1}\right)^2 + \psi \left(\pi_t + x_t - x_{t-1}\right)^2 ,$$

and the mapping with the delegation parameters in the general case is:

$$\theta_{xx} = \psi; 1 + \theta_{\pi\pi} = \alpha + \psi; \theta_{ss} = \psi + \alpha\delta^2; \theta_{\pi x} = \psi; \theta_{\pi s} = -\psi - \alpha\delta; \theta_{xs} = -\psi.$$

Substituting in (3.2) we find the optimal delegation parameters:

$$\begin{aligned} \alpha^* &= \frac{\lambda\chi^2}{\gamma^2\left(\chi - \beta\right)};\ \delta^* = \frac{\gamma}{\chi};\ \psi^* = \frac{\lambda\chi}{\gamma\left(\frac{\kappa}{\lambda} - 1\right)\left(\chi - \beta\right)} \\ \text{where } \chi &\equiv \left(\frac{\kappa}{\lambda} + 1\right)\gamma + 1 \end{aligned}$$

Note that $\psi^* > 0$ since, as argued in Section 1, the slope of the Phillips curve is larger than the welfare-based weight on output gap stabilization, $\kappa > \lambda$. Since $\theta_{\pi s}^* = -\psi^* - \alpha^*\delta^* = -\kappa\psi^*/\lambda < 0$, the second order condition is satisfied.

A similar delegation scheme would have the central bank facing a performance contract in addition to the nominal income growth objective:

$$L_t^b = \lambda x_t^2 + \alpha \pi_t^2 + \psi \left(\pi_t + x_t - x_{t-1}\right)^2 + 2c_\pi x_{t-1}\pi_t,$$

so that

$$\theta_{xx} = \psi; 1 + \theta_{\pi\pi} = \alpha + \psi; \theta_{ss} = \psi; \theta_{\pi x} = \psi; \theta_{\pi s} = c_\pi - \psi; \theta_{xs} = -\psi.$$

The optimal delegation parameters are:

$$\alpha^* = \frac{\lambda}{\gamma^2}\left[\left(\frac{\kappa}{\lambda} + 1\right)\gamma + 1\right];\ \psi^* = \frac{\lambda}{\gamma\left(\frac{\kappa}{\lambda} - 1\right)};\ c_\pi^* = -\frac{\lambda}{\gamma}$$

Similar reasoning as in the previous case ensures that $\psi^* > 0$ and hence that the second-order condition holds $\theta_{\pi s}^* < 0$.

[16] Hall and Mankiw (1999) also study the merits of this policy regime.

5. Conclusions

Central banks may have incentives to over-react to (cost-push) shocks of the type that create a trade-off between output and inflation stabilization, and the modern framework used for monetary policy analysis provides one justification for this bias. Specifically, a central bank operating under discretion does not have the type of inertial behavior that it would have if it were able to commit and take into account the effects of its current choices on future expected outcomes. This paper proposes a method to solve this commitment problem by optimal delegation: a central bank delegated with a different objective function may be able to replicate the commitment outcome despite operating under discretion. I outline a general method to solve for the optimal delegation parameters in a linear-quadratic framework and give a few examples of 'targeting regimes' that can be employed to that end. These include inflation and output gap contracts, as well as inflation, output gap growth and nominal income growth targeting, and are similar to delegation solutions proposed to solve other distortions in earlier, Barro-Gordon type models of monetary policy.

One important characteristic of optimal delegation is that none of the studied targeting regimes by itself can fully restore the optimum, but only specific combinations of them can do so. All delegation schemes considered here induce the optimal amount of inertia by giving a central bank operating under discretion the right incentives to *both* let an output gap contraction persist (e.g. by rewarding a future contraction, given a current contraction) *and* deflate (i.e. reward future deflation, given a current contraction). This can be viewed as providing support for a monetary policy strategy with multiple objectives, even though the social objective function features only price and output gap stability.

Two further features of these delegation schemes are worth emphasizing. First, insofar as the timeless-optimal commitment solution is time-consistent, delegation schemes that exactly replicate it are also time-consistent. More precisely, the government (or whoever is the ultimate principal that delegates monetary policy conduct) has no incentives to deviate from the optimal delegation scheme. This is in contrast to earlier delegation schemes purported to solve the 'average inflation bias' problem, which as McCallum (1995) and Jensen (1997) convincingly argued served merely to relocate the time-inconsistency problem at an earlier (delegation) stage: because the equilibrium that they are purported to implement is time inconsistent, so are the delegation schemes themselves.

Second, optimal delegation results in equilibrium determinacy: the equilibrium obtained under discretion by maximizing the objective function derived under optimal delegation is identical to that obtained under timeless-optimal commitment and is hence unique. This is important, because determinacy under pure discretion is not granted, and because commitment (or a substitute thereof) to the timeless-optimal targeting rule induces equilibrium determinacy even in cases where the underlying model would imply that standard determinacy results do not hold (e.g. under limited asset markets participation as in Bilbiie, 2007).

There are also two shortcomings that point to potentially fruitful areas for future research. First, the policy regimes for which we derived optimal delegation parameters are equivalent in the simple model presented here. However, it is likely that in more empirically realistic models of the transmission mechanism that build on this simple model (e.g. Christiano, Eichenbaum and Evans, 2005 and Smets and

Wouters, 2007) exact implementation of the timeless-optimal commitment solution will be impossible and the regimes studied here will have different welfare implications. A welfare-ranking of these regimes in such a model, and the investigation of other regimes allowed by our general delegation scheme is a potentially fruitful area for future research.

A related literature studies how central banks can achieve better outcomes than the discretionary one by resorting to reputational mechanisms (e.g. in Stokey,1989), informational imperfections (Backus and Driffill, 1985 and Canzoneri, 1985) or deviations from rational expectations (e.g.Taylor 1982). But as noted by Woodford (2003a), the outcome achieved by a central bank that is delegated 'optimally' (in the absence of such considerations), but inadvertently tries to achieve a 'better' equilibrium (by exploiting reputational mechanisms, informational imperfections, or building credibility) may be one that is suboptimal from a societal viewpoint.

References

[1] Backus, D. and Driffill, J., 1985. 'Inflation and reputation', *American Economic Review* 75, 530-538.

[2] Barro, R. J. and Gordon, D., 1983b. 'A Positive Theory of Monetary Policy in a Natural Rate Model,' *Journal of Political Economy*, 91(4), 589-610.

[3] Beetsma, R. and Jensen, H., 1999. 'Optimal inflation targets, 'conservative' central bankers and linear inflation contracts: Comment on Svensson (1997)', *American Economic Review* 89, 342-347.

[4] Bilbiie, F.O., 2007, 'Limited asset markets participation and inverted aggregate demand logic', *Journal of Economic Theory*, forthcoming.

[5] Blinder, A., 1997. '*Central banking in theory and practice*', Cambridge University Press.

[6] Canzoneri, M. B., 1985 'Monetary Policy Games and the Role of Private Information', *American Economic Review*, 75: 5, 1056-1070

[7] Christiano, L. Eichenbaum, M. and C. Evans, 2005, Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy*, 113:1

[8] Clarida, R., J. Galí, and M. Gertler, 1999, 'The science of monetary policy: a New Keynesian perspective', *Journal of Economic Literature*, 37, 1661-1707.

[9] Hall, R and N. G. Mankiw, 1994, "Nominal Income Targeting," in N. G. Mankiw, ed., '*Monetary policy*': University of Chicago Press: 71-93.

[10] Jensen, H., 1997. 'Credibility of optimal monetary delegation', *American Economic Review* 87, 911-920.

[11] Jensen, H., 2002. 'Targeting Nominal Income Growth or Inflation?', *American Economic Review* 92(4), 928-956.

[12] Kydland, F. and Prescott, E., 1977. 'Rules rather than discretion: the inconsistency of optimal plans', *Journal of Political Economy* 85, 473-490.

[13] Lockwood, B., 1997. 'State-contingent inflation contracts and unemployment persistence', *Journal of Money, Credit and Banking*, 29(3), 286-99.

[14] Lockwood, B.; Miller, M. and Zhang, L., 1998. 'Designing monetary policy when unemployment persists', *Economica* 65, 327-45.

[15] Lohmann, S., 1992, 'The optimal degree of commitment: credibility and flexibility' *American Economic Review*, 82, 273-286

[16] McCallum, B., 1995, 'Two fallacies concerning central-bank independence', *American Economic Review*, 85, 207-211

[17] McCallum, B. T. and E. Nelson, 2000, "Timeless Perspective vs. Discretionary Monetary Policy in Forward-Looking Models," NBER WorkingPapers No. 7915.

[18] Person, T. and G. Tabellini, 1993. "Designing Institutions for Monetary Stability." *Carnegie-Rochester Conference Series on Public Policy*, 39: 53-84.

[19] Rogoff, K., 1985, 'The optimal degree of commitment to a monetary target', *Quarterly Journal of Economics*, 100, 1169-90

[20] Smets F. and R. Wouters, 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3): 586-606,

[21] Stockey, N. 1989, 'Reputation and time consistency' *American Economic Review* Papers and Proceedings, 79: 134-139
[22] Svensson, L.E.O., 1997. 'Optimal inflation targets, conservative central bankers and linear inflation contracts', *American Economic Review*, 87-1, pp. 99-115
[23] Svensson, L.E.O., 1999a, "Inflation Targeting as a Monetary Policy Rule." *Journal of Monetary Economics*, 43(3): 607-54.
[24] Svensson, L.E.O., 1999b. 'Price level targeting vs. inflation targeting: a free lunch?' *Journal of Money, Credit and Banking* 31, 277–295.
[25] Taylor, J. B., 1982 'Establishing Credibility: A Rational Expectations Viewpoint.' *American Economic Review* 72: 81-85.
[26] Thompson, E., 1981, 'Who Should Control Money Supply?'*American Economic Review,* 71(2), 356-361.
[27] Vestin, D., 2006, 'Price-Level versus Inflation Targeting,' *Journal of Monetary Economics* 53: 1361–1376
[28] Walsh, C., 1995. 'Optimal Contracts for central bankers' *American Economic Review* 85(1), pp.150-67
[29] Walsh, C., 2003. 'Speed Limit Policies: The Output Gap and Optimal Monetary Policy' *American Economic Review* 93(1), pp.265-78
[30] Woodford, M., 1999, "Optimal Monetary Policy Inertia." National Bureau of Economic Research Working Paper No. 7261.
[31] Woodford, M., 2000, 'Pitfalls of Forward-Looking Monetary Policy,' *American Economic Review,* Papers and Proceedings, 90(2), 100-4.
[32] Woodford, M., 2003a, 'Optimal Interest-Rate Smoothing,' *Review of Economic Studies* 70, 861-886.
[33] Woodford, M., 2003b, '*Interest and Prices*' Princeton University Press.

Appendix A. Verification that optimal delegation leads to timeless-optimal commitment solution

The first order condition for the Markov perfect equilibrium under the optimal contract (2.2) is:

$$\frac{\lambda}{\kappa}\gamma\left(1+\frac{\beta\lambda}{\kappa\gamma+\lambda}\right)x_t+\gamma\pi_t-\frac{\lambda\gamma^2}{\kappa\gamma+\lambda}x_{t-1}-\frac{\lambda}{\kappa}\frac{\lambda\gamma}{\kappa\gamma+\lambda}x_{t-1}-\beta\frac{\lambda\gamma}{\kappa\gamma+\lambda}E_t\pi_{t+1}-\beta\frac{\lambda}{\kappa}\frac{\lambda\gamma}{\kappa\gamma+\lambda}E_tx_{t+1}=0,$$

which simplifies to:

$$\pi_t+\frac{\lambda}{\kappa}\left(x_t-x_{t-1}\right)=\beta\frac{\lambda}{\kappa\gamma+\lambda}E_t\left[\pi_{t+1}+\frac{\lambda}{\kappa}\left(x_{t+1}-x_t\right)\right].$$

But since $\beta\frac{\lambda}{\kappa\gamma+\lambda}<1$,this has as an unique solution

$$\pi_t+\frac{\lambda}{\kappa}\left(x_t-x_{t-1}\right)=0,$$

which is indeed identical to (1.6). The same is obtained for the problem of maximizing (3.1) when the delegation parameters satisfy (3.2).

Appendix B. On the importance of the second-order condition

This appendix shows the importance of the second-order condition by presenting two examples of delegation schemes in which the first-order condition holds, but the second-order condition fails. In the first example, the objective function implied by optimal delegation based on the first-order condition is concave rather than convex and hence identifies a maximum rather than a minimum. In the second, the objective is not strictly convex and hence implies indeterminacy - an infinity of equilibrium paths for output and inflation consistent with the first-order condition. Both cases have in common a focus on state-independent delegation.

Suppose first that the central bank is assigned a speed limit policy and a nominal income growth targeting objective:

$$L_t^b = \lambda_G \left(x_t - x_{t-1}\right)^2 + \alpha \pi_t^2 + \psi \left(\pi_t + x_t - x_{t-1}\right)^2 ,$$

which in terms of the general delegation parameters implies:

$\lambda + \theta_{xx} = \lambda + \lambda_G + \psi; 1 + \theta_{\pi\pi} = \alpha + \psi; \theta_{ss} = \lambda_G + \psi; \theta_{\pi x} = \psi; \theta_{\pi s} = -\psi; \theta_{xs} = -\lambda_G - \psi.$

Using the restrictions (3.2) found in by Proposition 2, we find the delegation parameters that replicate the first-order condition under timeless-optimal commitment:

$$\lambda_G = -\alpha \frac{\lambda}{\kappa};\ \psi = \alpha \frac{\lambda}{\kappa - \lambda};\ \alpha \text{ arbitrary}$$

For these parameter values, the loss function becomes: $L_t^b = \left[\pi_t + \frac{\lambda}{\kappa}\left(x_t - x_{t-1}\right)\right]^2$, suggesting a trivial, if not practically relevant optimal delegation scheme: the objective function should be the square of the first-order condition. However, on closer inspection we notice that this implies a violation of second-order condition (since, as argued above, $\kappa > \lambda$ and hence $\theta_{\pi s} > 0$). The first order condition holds, but it identifies a maximum rather than a minimum.

A different failure of the second-order condition occurs in another case of state-independent delegation $\theta_{\pi s} = \theta_{xs} = \theta_{ss} = 0$. These restrictions substituted in (3.2) imply:

$$\begin{aligned} \lambda + \gamma \theta_{\pi x} + \theta_{xx} &= 0 \\ \gamma \left(1 + \theta_{\pi\pi}\right) + \theta_{\pi x} &= 0 \end{aligned}$$

We need one more restriction since we have three unknowns and two equations, and we can choose to impose either $\theta_{xx} = 0$ or $\theta_{\pi\pi} = 0$, both of which deliver identical results. Imposing the latter delivers $\theta_{\pi x} = -\gamma, \lambda + \theta_{xx} = \gamma^2$, and the loss function becomes: $L_t^b = \left(\gamma x_t - \pi_t\right)^2$. It can be easily verified that for this loss function, the first-order condition for optimality is satisfied for any arbitrary pair of processes $\left(x_t, \pi_t\right)$; therefore, the model features an infinity of equilibria, which is only natural since the second-order condition fails due to $\theta_{\pi s} = 0$: the problem is convex, but not strictly convex.